6.3-6.6

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1 6.3, p.174

proof of Lemma 6.3.5 The authors use Lemma 6.3.3 to assert the existence of the Σ-formula $AxiomOfA(x)$. In Lemma 6.3.3, the Σ-formula in question is

$$\phi(v_1) :\equiv \exists z \exists y \exists c(Num(v_1, z) \wedge Sub(\overline{k}, \overline{8}, z, y) \wedge Deduction(c, y)).$$

Are they saying that this is what $AxiomOfA(x)$ is?
Also, is the replacement of the Δ-formula $AxiomOfN(e)$ with $AxiomOfA(e)$ the only change that turns $Deduction_A(c, v_1)$ into a Σ-formula?

2 6.4, p.182

Thm 6.4.5 Given Proposition 6.4.3, doesn't Theorem 6.4.5 follow from Propositions 6.4.3 and 6.4.4?

6.4.4: A is an ω-consistent and recursive set of axioms extending N
$\overset{6.4.3}{\Longrightarrow}$ A is a consistent and recursive set of axioms extending N
$\overset{\text{same as } 6.4.5}{\Longrightarrow}$ A is incomplete

3 6.5, p.185

p.185 Is this finite restriction of $\mathcal{L}_{NT}$ only used for "naming" the outline of the proof of Theorem 6.5.1?

p.186 In $Q \vdash ((\ \forall x)(\phi(x) \leftrightarrow x = \overline{n}))$, why is there a universal quantifier when it just ends up saying that x has to be some specific natural number n?

Do all formulas have to name a number? I'm tempted not to think so since the definition of "naming" seems to say (as in the last paragraph of the section) that a number named by a formula is the "number that makes the formula true." But surely there are formulas that are true or provable for not just one natural number.

Is "naming" used for anything other this one outline of proof? They really make it stand out but seem to use it for just this one thing.

4 6.6, p.187

p.191 At the end of the paragraph preceding Corollary 6.6.4, the authors seem to suggest that we're about to assume PA is inconsistent for the corollary, but then the corollary starts by saying "If PA is consistent,..." Is it the "...$\cup\{\neg Con_{PA}\}$" that says PA is inconsistent? So, the task at hand is to show $PA \cup \{\neg Con_{PA}\}$ is consistent?

5 Somewhat random questions

I'm just now remembering how in matrix algebra we say a system is inconsistent if the reduced echelon form has a row of zeros ending in a nonzero number. For instance,

$$\left[\begin{array}{cc|c} 3 & 27 & 9 \\ 2 & 18 & 7 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 9 & 3 \\ 2 & 18 & 7 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 9 & 3 \\ 0 & 0 & 1 \end{array}\right]$$

is an inconsistent system of equations because the last row is saying $0x+0y = 0 = 1$.

Does this directly parallel the discussion of consistency and inconsistency in this book? In other words, are we in a way taking $3x + 27y = 9$ and $2x + 18y = 7$ as axioms and seeing whether that set of axioms is consistent or not? Also, the authors have repeatedly said that if a set of axioms can prove $\perp$ (or any contradiction?) that we would then be able to prove anything, which certainly sounds like something that would result from $0 = 1$. But then again, I suppose it may just be borrowed verbiage, since this could be thought of as a regular old contradiction, but it just happened to cross my mind.

Apart from Chapter 7 and the works referenced throughout this book, do you have any other books you can recommend?