1.5

Andrew Lounsbury

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1 1.5, p.21

1. (a) $\underline{(\forall x)(\forall y)(x+y=2)}$
   free variables: none, so this is a sentence

   (b) $\underline{(x+y<x) \lor (\forall z)(z<0)}$
   free variables: x, y, so this is not a sentence

   (c) $\underline{((\forall y)(y<x)) \lor ((\forall x)(x<y))}$
   free variables: x, y, so this is not a sentence

6. $\phi(x) :\equiv \underbrace{[(\forall y)(x=y)]}_{\alpha} \lor \underbrace{[(\forall x)(x<0)]}_{\beta}$, $t :\equiv S0$

   The variable x is only free in α, so $\phi(t) :\equiv [(\forall y)(t=y)] \lor [(\forall x)(x<0)]$

2 1.6, p.26

3. $\mathcal{L}$ is $\{\flat, \sharp^3, \natural^2\}$

7. $\mathcal{L}_{NT}$ is $\{0, S, +, \cdot, E, <\}$
   $\mathfrak{N} = (\mathbb{N}, 0^{\mathfrak{N}}, S^{\mathfrak{N}}, +^{\mathfrak{N}}, \cdot^{\mathfrak{N}}, E^{\mathfrak{N}}, <^{\mathfrak{N}})$
   $S^{\mathfrak{N}}(t) :\equiv St$
   $+^{\mathfrak{N}}(t,s) :\equiv +ts$
   $\cdot^{\mathfrak{N}}(t,s) :\equiv \cdot ts$
   $E^{\mathfrak{N}}(t,s) :\equiv Ets$
   $<^{\mathfrak{N}}(t,s) :\equiv < ts$

   $S0 + S0 \stackrel{?}{=} SS0$

3 1.7, p.32

1. The structure $\mathfrak{N}$ makes the sentence $1 + 1 = 2$ true.

   Proof. ■

5. Let $\mathfrak{A}$ be a structure for the language of set theory, $\mathcal{L}_{ST}$, which is $\{\in\}$. Let $A = \{u, v, w, \{u\}, \{u, v\}, \{u, v, w\}\}$.

   $(\ \forall y \in y)(\exists x \in x)(x = y)$