6.2

Andrew Lounsbury

April 15, 2020

1 6.2, p.173

4. Let $\psi(v_1)$ be $Even(v_1)$, and let ϕ be the sentence generated when the Self-Reference Lemma is applied to $\psi(v_1)$. Then, $N \vdash \phi$, and $N \nvdash \neg\phi$.

   Proof. By the Self-Referencing Lemma, $N \vdash \phi \leftrightarrow Even(\overline{\ulcorner \phi \urcorner})$.
   $\ulcorner \phi \urcorner$ is a product of powers of the first few primes
   $\implies 2 \mid \ulcorner \phi \urcorner$

   The authors instead use the fact that ϕ is a sentence. I'm not sure why.

   $\implies N \vdash Even(v_1)$ since $Even(v_1)$ is a Δ-formula

   I know this step follows from Prop. 4.6.1 on p.115, but I don't think I've quite yet wrapped my head around the significance of Δ-formulas in all this. If $Even(v_1)$ were a Σ-formula, wouldn't this statement still be true by Prop. 4.6.2? And if $Even(v_1)$ were a Π-formula, then we wouldn't be able to say anything about whether $N \vdash \phi$ or $N \vdash \neg\phi$, right? HETZEL: So, the way to think about a Δ-formula is that all quantifiers are bounded. This gives rise to a powerful fact that N is strong enough to prove all true Δ-sentences **and** refute all false Δ-sentences. However, if $Even(v_1)$ were merely a Σ-formula, then N is strong enough to prove it if it were true, but not strong enough to refute it if it were false. And, worse yet, if $Even(v_1)$ were merely a Π-formula, then N is not strong enough to prove it if it were true **and** N is not strong enough to refute it if it were false. $\therefore N \vdash \phi$ and $N \nvdash \neg\phi$. ■