1.4

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1 1.4, p.17

1. The sum of the interior angles in a convex n-gon is $(n-2)\cdot 180°$.

   Proof. Base case:

   Induction step: Suppose a k-gon, where k is arbitrary, has interior angles that sum to $(k-2)\cdot 180°$. This k-gon has k interior angles. If we add another edge to form a $(k+1)$-gon, that $(k+1)$-gon has $k+1$ interior angles. If we pick a point on the $(k+1)$-gon and connect it to the other $k+1-3=k-2$ nonadjacent points, we split the $(k+1)$-gon into $k-1$ triangles whose angles together form the angles of the $(k+1)$-gon. Since each triangle's interior angles has $180°$, the sum of the $(k+1)$-gon's interior angles is $(k-1)\cdot 180° = ((k+1)-2)\cdot 180°$, as desired. ■

5. Let $\mathcal{L}$ be the language $\{0, <\}$.
   The number of symbols in any formula is divisible by 3.

   Proof. The atomic formulas here are $0=0$ and $0<0$, and each has 3 symbols. If α and β are atomic formulas with a symbols in α and b symbols in β, then $(\neg\alpha)$ has $a+3$ symbols, $(\alpha\vee\beta)$ has $a+b+3$ symbols and $(\forall v)(\alpha)$ has $a+6$ symbols. Since $3\mid a$ and $3\mid b$, we also have $3 \mid a+b+3, 3 \mid a+6$, and $3 \mid a+b+3$. Thus, the number of symbols is always divisible by 3. ■

6. If s and t are terms, then s is not an initial segment of t.

   Proof. **<u>Base case</u>:** If s is a variable or a constant symbol and t is as well, then when we write $t :\equiv su$, where u is a nonempty string, we know that u must be an empty, contradicting the assumption that s is an initial segment of t.
   In the case where $t :\equiv ft_1 \cdots t_n$ and f is a function symbol and $t_1, \ldots, t_n$ are terms, we would have to have $s :\equiv f$, where f is a nullary function symbol. However, this means u must be empty, a contradiction.
   is Now, suppose the term $s :\equiv ft_1 \cdots t_n$, is an initial segment of the term t. If t is a variable or a constant, then when we try to write $t :\equiv su$ for some nonempty string u, we must have $s :\equiv f$, where f is a nullary function symbol. This forces u to be an empty string, which is a contradiction.
   In the case where $t := gr_1 \cdots r_m$, where g is a function symbol and $r_1, \ldots, r_m$ are terms, we must have $f :\equiv g$ since they are the functions at the beginning of s and t, respectively. ■