5.2-5.5 Questions

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1 5.2

I didn't mark any questions for section 5.2.

2 5.3

p. 120 This is more of a comment, but $N \vdash \forall y[\phi(\overline{a}, y) \leftrightarrow y = \overline{b}]$ seems to me a bit reminiscent of well-definedness for functions. If N provides a deduction for $\phi(\overline{a}, y)$ for some $\mathcal{L}_{NT}$-term $\overline{a}$ and some y, then y must also be some $\mathcal{L}_{NT}$-term $\overline{b}$. Otherwise, the formula $\phi(\overline{a}, y)$ isn't really saying anything, much in the same way that a function that isn't well-defined can be somewhat nonsensical. (right?) HETZEL: The idea here is that a "traditional" function like $f(x) = x^2$ is given and we want to know whether there is a formula ϕ in the system that can precisely express or represent such a function. The answer is sometimes "yes" and sometimes "no". What is provided to you in the displayed statement is what we will mean by "represent".

p.120 In Definitions 5.3.1 through 5.3.4, they say representable **(in N)**, but they don't do this through the rest of the chapter. I'm assuming that's because they implicitly mean "in N" wherever it might be implied, but might it be because we could speak of representable/definable sets/functions in sets of axioms other than N? HETZEL: You got it.

p.120, Def. 5.3.1 So, for example, $\phi :\equiv (\forall x)x = x$ represents $A = \mathbb{N}$ (or any other A), which means all underlying sets are representable. (right? Or does ϕ

need to be a Δ-formula, specifically?) HETZEL: Yes, all underlying sets are representable, but of course not all subsets of those underlying sets are representable.

p.121, Def. 5.3.4 Here they seem to say that a function is a subset. Do they mean to say "Notice that a total function is representable if and only if it*'s codomain*(?) is a representable subset of $\mathbb{N}^{k+1}$." HETZEL: No. If we were talking about functions $f\colon \mathbb{N} \to \mathbb{N}$ (that is, $k = 1$), then such any such function f could be identified with a certain collection of ordered pairs in $\mathbb{N}^2$ given by $\{(a, b) \mid a \in \mathbb{N} \wedge b = f(a)\}$. This idea can be extended to $\mathbb{N}^{k+1}$ for functions $f\colon \mathbb{N}^k \to \mathbb{N}$. So, the way the book stated things is correct.

p.121 Def 5.3.5 Why do they say "(possibly)"? This is somewhat confusing. HETZEL: The reason they say "possibly" is because a representable function must be a total function, whereas a weakly representable function need not be, *but could be*. Since a total function requires a domain of $\mathbb{N}^k$ while a partial function requires a domain that is a *proper* subset of $\mathbb{N}^k$, the use of the term "possibly" is apt.

3 5.4

I didn't mark any questions for section 5.4.

4 5.5

p.135, Table 5.1 Is there any reason for using these numbers in particular? For instance, would it matter at all if ")" had the Symbol Number 23 while "(" had the Symbol Number 21? HETZEL: No, it would not matter. The only thing you're really shooting for is that each symbol is given a different number.