3.3 - 3.4

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1 3.3, p.93

1. $\Sigma :\equiv$ all $\mathcal{L}_{NT}$-formulas that are true in $\mathfrak{N}$
   $\phi :\equiv (\forall x)(\exists n)(x = \underbrace{SS \cdots S}_{nS\text{'s}} 0)$ is in Σ
   $\therefore$ There can be no nonstandard elements in any model of Σ.
   Show why this is fallacious.

   $\phi :\equiv$ "We can repeated apply S to 0 to obtain any x (<u>in the natural numbers</u>)"

   This argument doesn't take into account any models with a universe other than $\mathbb{N}$.

3. If $\mathfrak{A}$ and $\mathfrak{B}$ are $\mathcal{L}$-structures such that $\mathfrak{A} \cong \mathfrak{B}$, then $\mathfrak{A} \equiv \mathfrak{B}$.

   Proof. ■

5. Every nonstandard model of arithmetic contains an infinite prime number, that is, an infinite number a such that if $a = bc$, then either $b = 1$ or $c = 1$.

   Proof. ■

2 3.4, p. 101

1. Suppose that $\mathfrak{B} \subseteq \mathfrak{A}$, that ϕ is of the form $(\forall x)\psi$, where ψ is quantifier-free, and that $\mathfrak{A} \vDash \phi$. Then, $\mathfrak{B} \vDash \phi$.

Proof. $\mathfrak{A} \vDash \phi[s]$ with every s
$\mathfrak{A} \vDash (\ \forall x)\psi[s]$ with every s
$\mathfrak{A} \vDash \psi[s(x|a)]$ with every s and for every a in A
$\mathfrak{B} \vDash \psi[s(x|a)]$ with every s and for every a in A
$\mathfrak{B} \vDash (\ \forall x)\psi[s]$ with every s
$\mathfrak{B} \vDash \phi[s]$ with every s ■

Suppose that $\mathfrak{B} \subseteq \mathfrak{A}$, that ϕ is of the form $(\exists x)\psi$, where ψ is quantifier-free, and that $\mathfrak{A} \vDash \phi$. Then, $\mathfrak{B} \vDash \phi$.

Proof. $\mathfrak{A} \vDash \phi[s]$ for every s
$\mathfrak{A}(\exists x)\psi[s]$ for every s
$\mathfrak{A}\neg(\ \forall x) \neq \psi[s]$ for every s
$\mathfrak{A} \nvDash (\ \forall x) \neq \psi[s]$ for every s
$\mathfrak{A} \nvDash \neg\psi[s(x|a)]$ for every s and every a in A
$\mathfrak{A} \vDash \psi[s(x|a)]$ for every s and every a in A ■

8.