AN ABSTRACT OF A THESIS

AN ANALYSIS OF THE VIBRATION OF TRANSVERSELY ISOTROPIC TOROIDAL SHELLS

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The purpose of this study was to analyze an open cross section transversely isotropic toroidal shell having a hollow circular cross section through the use of the 'torsvib1dp' FORTRAN program. The method of analysis was a numerical method developed by Buchanan and Liu [1], which was based upon the general three-dimensional theory of dyadic elasticity. The strain-displacement equations and the equations of motion were derived in toroidal coordinates. The displacement functions were given in terms of the circular frequency. These equations were combined to produce the toroidal governing equations of motion, which were assumed to be axisymmetric for simplification. The nine-node Lagrangian finite element was formulated based on the toroidal coordinate system. The standard eigenvalue problem for vibrations was solved to determine the natural frequencies and the corresponding mode shapes for various shell configurations and material properties.

The shell configurations considered in this study included two geometric parameters. These parameters were the thickness ratio, which was the ratio of the shell thickness, t, to the cross-sectional radius, r, and the radial ratio, which was the ratio of the cross-sectional radius, r, to the radius of revolution, R. Two values of the thickness ratio have been examined and these were 1/10 and 1/20. These values were to represent the geometry of a moderately thick shell and a thin shell, respectively. Along with each of these, three values of the radial ratio were evaluated for each case. These quantities of the radial ratio were 1/4, which was to represent geometry similar to that of an automobile tire, 1/6, which was intended to represent an aircraft tire, and 1/10, which was intended to represent a motorcycle tire. Along with each geometric parameter, various material properties were also considered. The material properties included isotropic rubber, rubber with reinforcement about the major axis, rubber with reinforcement about the minor axis, and rubber with reinforcement about both axes. Two types of reinforcement materials were considered, these were epoxy and steel cords. The first five natural frequencies in the plane of the cross section were determined for each case and the corresponding mode shapes were plotted. The results have been validated using ANSYS and were found to be in very close agreement.