

Tennessee Technological University
Mathematics Department

MATH 4010-4020/5010-5020: Modern Algebra I-II

I. COURSE DESCRIPTION FROM CATALOG: Lec. 3-3. Cr. 3-3.

Groups and subgroups including cyclic, abelian, finite; permutation groups, group homomorphisms, cosets and Lagrange's Theorem, normal subgroups and factor groups. Rings including integral domains, unique factorization domains and Euclidean domains, ideals and factor rings, ring homomorphisms, fields and their extensions, geometric constructions.

II. PREREQUISITE(S):

MATH 4010/5010: C or better in MATH 2010 and C or better in MATH 3400.

MATH 4020/5020: C or better in MATH 4010/5010.

III. COURSE OBJECTIVE(S):

To enable the student to obtain a broad overview of the most common algebraic systems and to begin attaining a working knowledge of groups, rings, fields, and integral domains.

IV. TOPICS TO BE COVERED:

(4010/5010) Chapter 1 - Integers

1. Divisors
2. Primes
3. Congruences
4. Integers Modulo n

Chapter 2 - Functions

1. Functions
2. Equivalence Relations
3. Permutations

Chapter 3 - Groups

1. Definition of a Group
2. Subgroups
3. Constructing Examples
4. Isomorphisms
5. Cyclic Groups
6. Permutation Groups
7. Homomorphisms
8. Cosets, Normal Subgroups, and Factor Groups

Chapter 7 - Structure of Groups

1. Isomorphism Theorems; Automorphisms
2. Conjugacy (As time permits)
3. Groups Acting on Sets (As time permits)
4. The Sylow Theorems (As time permits)
5. Finite Abelian Groups (As time permits)
6. Solvable Groups (As time permits)
7. Simple Groups (As time permits)

(4020/5020) Chapter 4 - Polynomials

1. Fields; Roots of Polynomials
2. Factors
3. Existence of Roots
4. Polynomials over \mathbb{Z} , \mathbb{Q} , \mathbb{R} , and \mathbb{C}

Chapter 5 - Commutative Rings

1. Commutative Rings; Integral Domains
2. Ring Homomorphisms
3. Ideals and Factor Rings
4. Quotient Fields

Chapter 9 - Unique Factorization

1. Principal Ideal Domains
2. Unique Factorization Domains
3. Some Diophantine Equations (As time permits)

Chapter 6 - Fields

1. Algebraic Elements
2. Finite and Algebraic Extensions
3. Geometric Constructions
4. Splitting Fields
5. Finite Fields
6. Irreducible Polynomials over Finite Fields
7. Quadratic Reciprocity (As time permits)

Chapter 8 - Galois Theory

1. The Galois Group of a Polynomial (As time permits)
2. Multiplicity of Roots (As time permits)
3. The Fundamental Theorem of Galois Theory (As time permits)
4. Solvability by Radicals (As time permits)
5. Cyclotomic Polynomials (As time permits)
6. Computing Galois Groups (As time permits)

V. ADDITIONAL INFORMATION:

Graduate credit is earned on the basis of additional work required by the instructor [per 2005-2006 TTU Graduate Bulletin], page 38.

VI. POSSIBLE TEXTS AND REFERENCES:

Abstract Algebra, 3rd ed., Beachy and Blair
Contemporary Abstract Algebra, 7th ed., Gallian
Modern Algebra: An Introduction, 4th ed., Durbin
Schaum's Outline of Modern Abstract Algebra, Ayers

VII. ANY TECHNOLOGY THAT MAY BE USED:

Maple