Tennessee Technological University
Mathematics Department

MATH 2010: Introduction to Linear Algebra

I. COURSE DESCRIPTION FROM CATALOG:

Systems of linear equations, matrix algebra, inverses, matrix factorizations, determinants, vector spaces and dimension, rank, linear transformations, eigenvalues and eigenvectors, inner product, orthogonal projections
Lec. 3. Cr. 3.

II. PREREQUISITE(S):

C or better in MATH 1910

III. COURSE OBJECTIVE(S):

To introduce topics in matrix algebra to mathematics, science and engineering students along with appropriate applications.

IV. STUDENT LEARNING OUTCOMES:

Upon successful completion of the course students will understand the concepts of a vector space, subspace, dimension, rank, basis, and linear transformation; perform basic matrix algebra operations, including finding the inverse and the determinant, to solve systems of linear equations and to understand when a system is consistent or inconsistent; understand the connection between linear transformations and matrices and determine the null space, column space, eigenvectors, eigenvalues, and eigenspaces of a linear transformation/matrix; and use basic operations on a real vector space to determine an orthonormal basis for the space via the Gram-Schmidt orthonormalization process.

V. LIST OF SPECIFIC CONCEPTS WITH TOPICS:

Concept 1: Matrices, Matrix Operations, and Matrix Algebra:

(a) Definition and examples of matrices with real and complex entries including but not limited to the following types: augmented, diagonal, upper triangular, lower triangular, symmetric, antisymmetric, Hermitian, anti-Hermitian, elementary, permutation, row-scaling, row-reduced, in echelon form, in reduced echelon form, orthogonal, unitary, nilpotent, idempotent, identity.
(b) Matrix addition, scalar multiple, transposition with properties, Hermitian conjugation with properties.
(c) Matrix algebra including: associativity of matrix multiplication, general non-commutativity but knowing examples of matrices that do commute, distributive laws, definition of inverse of a matrix, uniqueness of inverse, conditions guaranteeing existence of the inverse, matrix rank, matrix determinant, algorithm to compute the matrix inverse A^{-1}.
(d) Three ways to look at matrix multiplication: row picture, column picture, row-column picture.
(e) Rank and Nullity Theorem
(f) Matrix factorizations such as: *LU*-factorization (leave *QR*-factorization, *SVD*-decomposition as optional)

Concept 2: Systems of linear equations: definition, solving, etc:

(a) Augmented matrix [A | **b**] of a linear system, homogeneous and non-homogeneous, free and basis variables, overdetermined system, undetermined system, consistent system, inconsistent system
(b) Elementary row operations: row switching, row scaling, row addition, forward (Gaussian) elimination, back substitution, backward (Jordan) elimination, pivots.
(c) Vector form A **x** = **b** of a linear system and writing solution to a consistent system in vector form **x** as a sum of a homogeneous solution $\mathbf{x}_h$ and a particular solution $\mathbf{x}_p$
(d) Existence of Solution to a linear system when rank(A) = rank([A | **b**]) and Uniqueness of Solution when rank(A) = rank([A | **b**]) = n where n is the number of indeterminates or the number of columns in A.
(e) *LU*-factorization of A, that is, A = LU = LDU', possibly PA = LU = LDU' via elementary matrices (optional).
(f) Using A = LU to efficiently solve A**x** = **b** provided rank(A)=rank([A | **b**]).

Concept 3: Vectors and vector spaces including:

(a) Row vectors and column vectors, scalar product
(b) Spanning, linear dependence/independence, basis, dimension
(c) Definition of a vector space and subspace: examples of such including $\mathbf{R}^m$, $\mathbf{C}^m$, $\mathbf{R}^{m\times n}$,
(d) At least two Fundamental Spaces of any matrix: Null Space and Column Space including their definitions, bases and dimensions (leave Row Space and Left Null Space as optional)

Concept 4: Linear transformations between vector spaces:

(a) Kernel, range, nullity, rank, Rank and Nullity (finite dimensional case)
(b) Matrix [T]_{B,B'} of a linear transformation T: V → W with respect to chosen bases B and B' in V and W, respectively.
(c) (Optional) Change of basis matrix and relation between $[T]_B$ and $[T]_{B'}$ for T:V → V and two bases B and B' in V.
(d) Connection between composition T ∘ S of linear transformations and matrix product of their matrices with respect to some bases.

Concept 5: Determinant and its properties:

(a) Definition and properties of matrix determinant det(M);
(b) Computation of det(A) via Laplace expansion or via the *LU*-factorization or through an echelon form (up to a scalar), Cramer's Rules (at least in 2×2 case).

Concept 6: Eigenvalues and eigenvectors:

(a) Definition of eigenvalue λ and eigenvector **x** of a matrix A as A**x** = λ**x**, **x** ≠ 0.
(b) Characteristic polynomial of a matrix and Cayley-Hamilton Theorem
(c) Algebraic and geometric multiplicities of an eigenvalue λ, eigenspace E_λ and its dimension,
(d) Eigenvectors corresponding to different eigenvalues are linearly independent
(e) Complete set of linear independent eigenvectors and eigenbasis for $\mathbf{R}^n$
(f) Problem of diagonalization of a square matrix with applications

Concept 7: Orthogonality and Gram-Schmidt orthogonalization process:

(a) Orthogonal and orthonormal vectors and basis, inner product;

(b) Gram-Schmidt orthogonalization process
(c) Projections on lines and subspaces,
(d) Least squares solutions to inconsistent systems
(e) *QR*-factorization process, normal system of equations (optional)

VI. ADDITIONAL INFORMATION:

VII. POSSIBLE TEXTS AND REFERENCES:

Linear Algebra and its Applications, 5th Edition, Lay (2011) (ISBN-10: 0-321-38517-9, ISBN-13 978-0-321-38517-8) (recommended)

VIII. ANY TECHNOLOGY THAT MAY BE USED:

Matlab (Octave) or Maple

IX. STUDENT ACADEMIC MISCONDUCT POLICY

Maintaining high standards of academic integrity in every class at Tennessee Tech is critical to the reputation of Tennessee Tech, its students, alumni, and the employers of Tennessee Tech graduates. The Student Academic Misconduct Policy describes the definitions of academic misconduct and policies and procedures for addressing Academic Misconduct at Tennessee Tech. For details, view the Tennessee Tech's Policy 217 – Student Academic Misconduct at Policy Central.

X. DISABILITY ACCOMMODATION

Students with a disability requiring accommodations should contact the Office of Disability Services (ODS). An Accommodation Request (AR) should be completed as soon as possible, preferably by the end of the first week of the course. The ODS is located in the Roaden University Center, Room 112; phone 372-6119. For details, view the Tennessee Tech's Policy 340 – Services for Students with Disabilities at Policy Central.