

**Tennessee Technological University  
Mathematics Department**

**MATH 4530-4540/5530-5540: Linear Algebra I-II**

**I. COURSE DESCRIPTION FROM CATALOG:**

A theoretical study of vector spaces, bases and dimension, subspaces, linear transformations, dualspaces, eigenvalues and eigenvectors, inner product spaces, spectral theory, duality, quadratic and bilinear forms. Lec. 3-3. Credit 3-3.

**II. PREREQUISITE(S):**

MATH 4530 (5530): C or better in MATH 2010 and MATH 3400.

MATH 4540 (5540): C or better in MATH 4530 or 5530.

**III. COURSE OBJECTIVE(S):**

To introduce students to the theory of linear operators on (mostly) finite dimensional real and complex vector spaces.

**IV. STUDENT LEARNING OUTCOMES:**

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Upon successful completion of the course students will understand the concepts of vector space, subspace, basis, eigenvector, eigenvalue, eigenspace, and linear transformation in an abstract context; understand the concepts of quotient space, direct sum, and isomorphism; and create proofs of simple results concerning the aforementioned concepts.

MATH 4540/5540

Upon successful completion of the course students will understand the concepts of an inner product space, orthonormal basis, orthogonal projection, and characteristic polynomial in an abstract context; understand the concepts of self-adjoint operator, normal operator, positive operator, isometry, generalized eigenvector, and minimal polynomial; understand and use the Gram-Schmidt Orthonormalization Procedure, Spectral Theorem, and Jordan canonical form to answer certain questions regarding vector spaces; and create proofs of simple results concerning the aforementioned concepts.

**V. TOPICS TO BE COVERED:**

**MATH 4530/5530:**

**Chapter 4: Vector Spaces** – Definition – Linear combinations, spanning sets – Subspaces – Linear spans, Row space of a matrix – Linear dependence and independence – Basis and dimension – Rank of a matrix – Sums and direct sums – Coordinates

**Chapter 5: Linear Mappings** – Definition – Kernel and image of linear mapping – Singular and nonsingular linear mappings – Isomorphisms – Operations with linear mappings – Algebra of linear operators

**Chapter 6: Linear Mappings and Matrices** – Matrix representation of a linear operator – Change of basis – Similarity – Matrices and general linear mappings

**Chapter 7: Inner Product Spaces, Orthogonality** – Definition – Examples of inner product spaces – Cauchy-Schwarz inequality – Applications – Orthogonality – Orthogonal sets and bases – Gram-Schmidt orthogonalization process – Orthogonal and positive definite matrices – Complex inner product spaces – Normed vector spaces

**Chapter 8: Determinants** – Definition – Properties – Minors and cofactors – Laplace expansion – Applications – Submatrices – Block matrices and determinants – Determinants and volume – Determinant of a linear operator – Multilinearity and determinants

#### **MATH 4540/5540:**

**Chapter 9: Diagonalization: Eigenvalues and Eigenvectors** – Polynomials of matrices – Characteristic polynomial and Cayley-Hamilton Theorem – Diagonalization, eigenvalues and eigenvectors, diagonalizing matrices – Diagonalizing real symmetric matrices and quadratic forms – Minimal polynomial – Characteristic and Minimal polynomials of block matrices

**Chapter 10: Canonical Forms** – Triangular form – Invariance – Invariant direct-sum decompositions – Primary decomposition – Nilpotent operators – Jordan canonical form – Cyclic subspaces – Rational canonical form – Quotient spaces

**Chapter 11: Linear Functionals and the Dual Space** – Linear functionals and the dual space – Dual basis – Second dual space – Annihilators – Transpose of a linear operator

**Chapter 12: Bilinear, Quadratic, and Hermitian Forms** – Bilinear forms and matrices – Alternating bilinear forms – Symmetric bilinear forms and quadratic forms – Real symmetric bilinear forms and Law of Inertia – Hermitian forms

**Chapter 13: Linear Operators on Inner Product Spaces** – Adjoint operators – Special linear operators – Self-adjoint operators – Orthogonal and unitary operators – Orthogonal and unitary matrices – Change of orthonormal basis – Positive definite and positive operators Diagonalization and canonical forms in inner product spaces – Spectral Theorem

#### **VI. ADDITIONAL INFORMATION:**

Graduate credit is earned on the basis of additional work that can be required by the instructor per 2005-2006 TTU Graduate Bulletin, page 38.

**VII. POSSIBLE TEXTS AND REFERENCES:**

*Advanced Linear Algebra*, 3<sup>rd</sup> edition by Roman  
*Schaum's Outline of Linear Algebra*, 4th edition, Lipschutz  
*Linear Algebra with Applications*, J.T. Scheick  
*Linear Algebra Done Right*, 2nd Edition by Axler  
*Advanced Linear Algebra*, 2nd edition, by Roman

**VIII. ANY TECHNOLOGY THAT MAY BE USED:**

**IX. STUDENT ACADEMIC MISCONDUCT POLICY:**

Maintaining high standards of academic integrity in every class at Tennessee Tech is critical to the reputation of Tennessee Tech, its students, alumni, and the employers of Tennessee Tech graduates. The Student Academic Misconduct Policy describes the definitions of academic misconduct and policies and procedures for addressing Academic Misconduct at Tennessee Tech. For details, view the Tennessee Tech's Policy 217 – Student Academic Misconduct at [Policy Central](#).

**X. DISABILITY ACCOMMODATION:**

Students with a disability requiring accommodations should contact the Office of Disability Services (ODS). An Accommodation Request (AR) should be completed as soon as possible, preferably by the end of the first week of the course. The ODS is located in the Roaden University Center, Room 112; phone 372-6119. For details, view the Tennessee Tech's Policy 340 – Services for Students with Disabilities at [Policy Central](#).