Tennessee Technological University
Mathematics Department

MATH 6110-6120: Abstract Algebra I-II

I. COURSE DESCRIPTION FROM CATALOG: An extensive treatment of groups, semigroups, integral domains, rings and ideals, fields and Galois fields. Lec. 3. Cr. 3.

II. PREREQUISITE(S):
MATH 6110: C or better in MATH 4010/5010 or consent of instructor.
MATH 6120: C or better in MATH 4020/5020 and C or better in MATH 6110, or consent of instructor.

III. COURSE OBJECTIVE(S):
To expand in-depth and breadth upon one-year undergraduate sequence in Abstract Algebra. New topics normally not covered in undergraduate sequences could include matrix algebras, free groups, generators and relations, bilinear forms, spectral theorem for normal operators, examples of algebras such as Clifford algebras, group algebras, Hecke algebras, or Lie algebras, linear groups, topics from group representations, algebraic geometry, quadratic fields, prime ideals, modules, the Structure Theorem for Abelian Groups, field extensions, Galois theory.

IV. TOPICS TO BE COVERED:

MATH 6110:

1. Groups: definition, examples, subgroups, isomorphisms, homomorphisms, equivalence relations and partitions, cosets, restriction of homomorphisms to a subgroup, products of groups, quotient groups.
2. Vector spaces over fields: definition, bases, dimensions, direct sums.
3. Linear transformations: kernel, image, the dimension formula, matrix of a linear transformation, characteristic and minimal polynomials, eigenvalues/eigenvectors, diagonalization, matrix exponentiation.
4. Symmetry: symmetry of plane figures, the group of motions of the plane, finite groups of motions, discrete groups of motions, operations on cosets, the Counting Formula, permutation representations.
5. More group theory: automorphisms, the Sylow Theorems, the free group, generators and relations, the Todd-Coxeter Algorithm.

Optional selected topics, if time permits:
6. Bilinear Forms.
7. Linear Groups.
8. Group Representations.

MATH 6120:

9. Rings: definition, examples, polynomial rings, homomorphisms, ideals, quotient rings, integral domains, fraction fields, maximal ideals.

10. Factorization: factorization of integers and polynomials, unique factorization domains, principal ideal domains, Euclidean domains, Gauss's Lemma, Gaussian integers, algebraic integers, quadratic fields, ideal factorization.
11. Modules: definitions, examples, free modules, bases, generators and relations for modules, the Structure Theorem for Abelian Groups, free modules over polynomial rings.
12. Fields: examples, algebraic and transcendental elements, field extensions, ruler and compass constructions, adjunction of roots, finite fields, transcendental extensions.
13. Galois Theory: the Main Theorem, cubic equations, symmetric functions, primitive elements, proof of the Main Theorem, quartic equations, Kummer extensions, cyclotomic extensions.

V. ADDITIONAL INFORMATION:

Lectures, homework, possibly lab assignments

VI. POSSIBLE TEXTS AND REFERENCES:

Abstract Algebra, Dummit and Foote, 3rd edition
Algebra by Michael Artin, Prentice Hall, Englewood Cliffs
A First Course in Abstract Algebra (2nd Edition) by Joseph J. Rotman, Prentice Hall, 2nd edition (February 28, 2000)
Advanced Modern Algebra by Joseph J. Rotman, Publisher: Prentice Hall, 1st edition (April 2002)

VII. ANY TECHNOLOGY THAT MAY BE USED: