Tennessee Technological University
Mathematics Department

MATH 6240-6250: Representations and Characters of Groups I and II (cr. 3, cr. 3)

I. COURSE DESCRIPTION FROM CATALOG:

MATH 6240:
FG-modules, reducibility, group algebras, FG-homomorphisms, Maschke's Theorem, Schur's Lemma, irreducible modules, characters, inner products of characters, character tables, orthogonality relations.
MATH 6250:
Normal subgroups and lifted characters, tensor products, restriction to a subgroup, induced modules and characters, Frobenius reciprocity relation, applications to group theory such as real representations, groups of order pq, p-groups, characters of GL(2,q), symmetric groups, Burnside's Theorem, and molecular vibrations. Lec. 3-3. Cr. 3-3.

II. PREREQUISITE(S):

MATH 6240: C or better in MATH 4010/5010 while C or better in MATH 4530/5530 is recommended, or consent of instructor

MATH 6250: C or better in MATH 6240

III. COURSE OBJECTIVE(S):

The main purpose of this sequence is to introduce advanced senior undergraduates and graduate students to a modern presentation of the representation theory of finite groups in terms of modules. Students will learn how to construct character tables of many groups of orders less than 32 and some simple groups of larger orders. Some of the examples will include but will not be limited to permutation groups, alternating groups, dihedral groups, and direct product groups. Students will learn how to construct reduced and induced representations. Representation theory can be then applied to algebraic integers and group theory to compute character tables of groups of order pq, some p-groups, symmetric groups, character table of GL(2,q), or prove, for example, famous Burnside Theorem that all groups of order $p^a q^b$, p and q prime numbers, a and b non-negative integers with a+b >= 2, are not simple. Applications of the representation theory extend to theoretical physics and chemistry. Examples covered in class may include normal modes of molecular vibrations.

This sequence will be accessible to students who have completed at least the first undergraduate course in group theory or, better, the first graduate course in group theory. It is recommended that students have completed at least one semester of matrix algebra including linear transformations or, better, the first semester of junior/senior level linear algebra. However, the necessary background in group theory and linear algebra will be briefly reviewed at the beginning of the sequence. The main approach to the representation theory as presented in this sequence is via group modules. Practical aspects of the subject are emphasized and the course abounds in examples and computations related to a variety of finite groups appearing in mathematics, physics and chemistry. Examples may also include character tables of all groups of order less than 32 as well as those of all p-groups of order at most p^4.

IV. STUDENT LEARNING OUTCOMES:

- Learning Outcomes for 6240: Upon successful completion of this course, students will understand the representation theory of finite groups such as permutation groups, alternating groups, dihedral groups and groups expressed as direct products, in terms of modules; be able to construct character tables of groups; be familiar with key theorems such as Maschke's Theorem and Schur's Lemma.
- Learning Outcomes for 6250: Upon successful completion of this course, students will understand tensor products; be able to construct character tables of groups of order pq, of p-groups, and of GL(2, q); be familiar with Burnside's Theorem

V. TOPICS TO BE COVERED:

MATH 6240:

1. Groups, homomorphisms, vector spaces and linear transformations
2. Group representations
3. FG-modules, submodules, and reducibility
4. Group algebras and FG-homomorphisms
5. Maschke’s Theorem and Schur’s Lemma
6. Irreducible modules and decompositions of group algebras
7. Conjugacy classes and characters
8. Inner products of characters
9. Irreducible characters
10. Character tables and orthogonality relations.

MATH 6250:

1. Normal subgroups and lifted characters
2. Elementary character tables
3. Tensor products
4. Restriction to a subgroup
5. Induced modules and characters
6. Algebraic integers (optional)
7. Real representations (optional)
8. Characters of groups of order pq (optional)
9. Characters of some p-groups (optional)

10. Character table of GL(2,q) (optional)
11. Permutations and characters
12. Applications to group theory including Burnside Theorem (optional)
13. Applications to physics and chemistry. (optional)

VI. ADDITIONAL INFORMATION:

The minimum theoretical material for this sequence comes from chapters 1—21 and 24 from James and Liebeck's book. It is desirable and possible to include in MATH 6250 the following additional topics: Chapter 25 (Characters of groups of order pq), Chapter 26 (Characters of some p-groups), and Chapter 29 (Permutations and characters). Depending on students' interests and the instructor interest, one can include applications of representation theory to molecular vibrations (Chapter 32), to algebraic integers (Chapter 22), real representations (Chapter 23), character table of GL(2,q) (Chapter 28), or a proof of Burnside's Theorem (Chapter 31).

VII. POSSIBLE TEXTS AND REFERENCES:

Primary:

1. Gordon James and Martin Liebeck, ``Representations of Characters of Groups", Cambridge University Press, 2nd edition (2001), Cambridge, New York, 2007. ISBN: 978-0521-81205-4 Hardback, 987-0-521-003392-6 Paperback

Supplementary:

2. Bruce E. Sagan, ``The Symmetric Group: Representations, Combinatorial Algorithms, and Symmetric Functions", Springer, 2nd edition (2001) ISBN: 0-387-95067-2
3. Morton Hamermesh, ``Group Theory and Its Application to Physical Problems", Dover Publications, New York (1962), ISBN: 0-486-66181-4

VIII. ANY TECHNOLOGY THAT MAY BE USED:

SymGroupAlgebra package for Maple for computations in group algebras of symmetric groups

IX. STUDENT ACADEMIC MISCONDUCT POLICY

Maintaining high standards of academic integrity in every class at Tennessee Tech is critical to the reputation of Tennessee Tech, its students, alumni, and the employers of Tennessee Tech graduates. The Student Academic Misconduct Policy describes the definitions of academic misconduct and policies and procedures for addressing Academic Misconduct at Tennessee Tech. For details, view the Tennessee Tech's Policy 217 – Student Academic Misconduct at Policy Central.

X. DISABILITY ACCOMMODATION

Students with a disability requiring accommodations should contact the Accessible Education Center (AEC). An Accommodation Request (AR) should be completed as soon as possible, preferably by the end of the first week of the course. The AEC is located in the Roaden University Center, Room 112; phone 931-372-6119. For details, view the Tennessee Tech's Policy 340 – Services for Students with Disabilities at Policy Central.