

CHAPTER 7

CONTROL OF FOUR LEGGED ACTIVE POWER FILTER WITH UNBALANCED AND NONLINEAR LOADS.

7.1 Modeling of 4- legged Active Power Filter in a-b-c Reference Frame

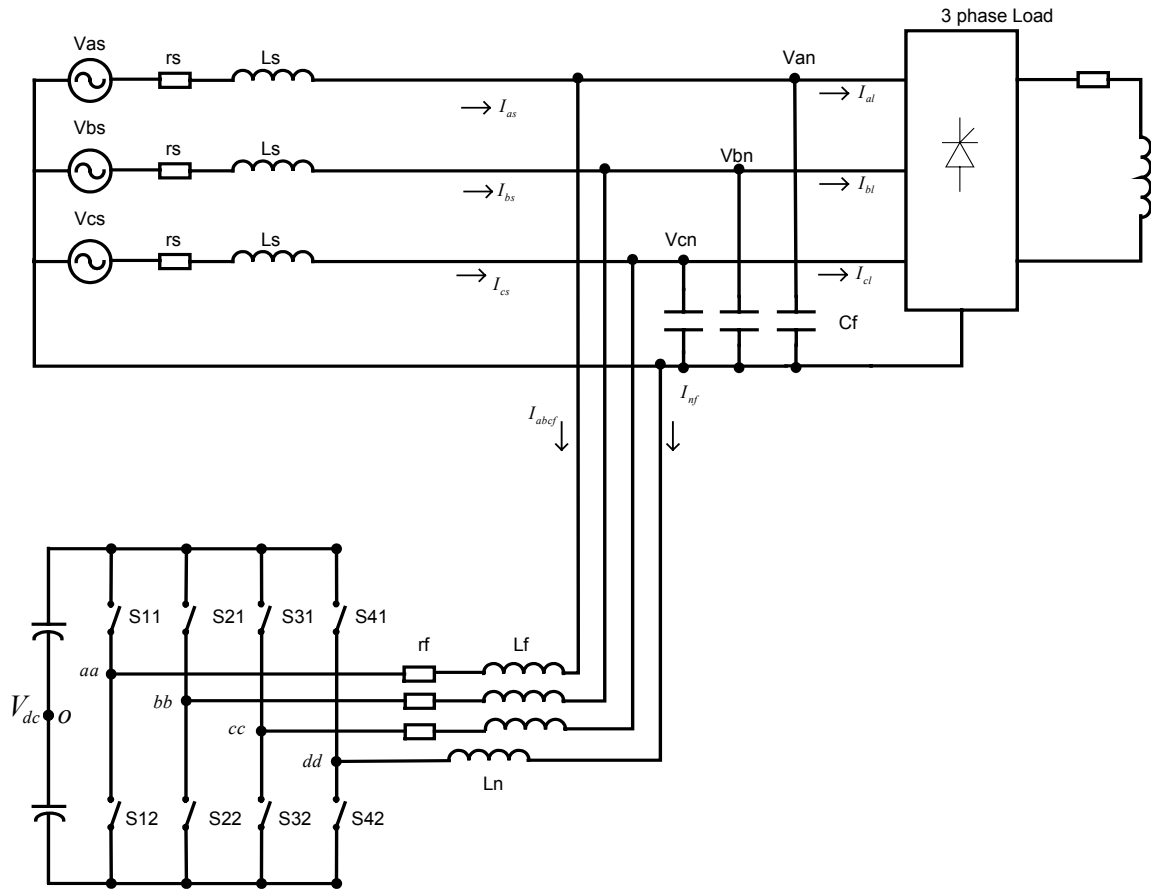


Figure 7.1 Modeling of 4- legged Active Power Filter.

The active power system can be modeled in a-b-c reference frame as follows.

The equations of source feeding the load are given as

$$V_{as} = r_s I_{as} + L_s p I_{as} + V_{an}$$

$$\begin{aligned}
 V_{bs} &= r_s I_{bs} + L_s p I_{bs} + V_{bn} \\
 V_{cs} &= r_s I_{cs} + L_s p I_{cs} + V_{cn}.
 \end{aligned} \tag{7.1}$$

As shown in the figure, writing the KVL equations across the loop from point 'a, b, c, and d', the output voltages of the active filter can be expressed as

$$\begin{aligned}
 V_{aao} &= V_{aan} + V_{no} \\
 V_{bbo} &= V_{bbn} + V_{no} \\
 V_{cco} &= V_{ccn} + V_{no} \\
 V_{ddo} &= V_{ddn} + V_{no}
 \end{aligned} \tag{7.2}$$

where

$$\begin{aligned}
 V_{aao} &= \frac{V_{dc}}{2} (S_{11} - S_{12}) \\
 V_{bbo} &= \frac{V_{dc}}{2} (S_{21} - S_{22}) \\
 V_{cco} &= \frac{V_{dc}}{2} (S_{31} - S_{32}) \\
 V_{ddo} &= \frac{V_{dc}}{2} (S_{41} - S_{42}).
 \end{aligned} \tag{7.3}$$

Now, the voltage loop equation is across the output is given as

$$\begin{aligned}
 V_{aan} &= Z_1 I_{af} + V_{an} \\
 V_{bbn} &= Z_1 I_{bf} + V_{bn} \\
 V_{ccn} &= Z_1 I_{cf} + V_{cn} \\
 V_{ddn} &= -Z_n I_{nf}.
 \end{aligned} \tag{7.4}$$

Substituting (7.3) and (7.4) in (7.2)

$$\frac{V_{dc}}{2}(S_{11} - S_{12}) = -Z_1 I_{af} + V_{an} + V_{no}$$

$$\frac{V_{dc}}{2}(S_{21} - S_{22}) = -Z_1 I_{bf} + V_{bn} + V_{no}$$

$$\frac{V_{dc}}{2}(S_{31} - S_{32}) = -Z_1 I_{cf} + V_{cn} + V_{no}$$

$$\frac{V_{dc}}{2}(S_{41} - S_{42}) = -Z_n I_{nf} + V_{no} \quad (7.5)$$

or $V_{no} = \frac{V_{dc}}{2}(S_{41} - S_{42}) + Z_n I_{nf}$, thus substituting V_{no} in (7.5)

$$\frac{V_{dc}}{2}(S_{11} - S_{12}) = -Z_1 I_{af} + V_{an} + \frac{V_{dc}}{2}(S_{41} - S_{42}) + Z_n I_{nf}$$

$$\frac{V_{dc}}{2}(S_{21} - S_{22}) = -Z_1 I_{bf} + V_{bn} + \frac{V_{dc}}{2}(S_{41} - S_{42}) + Z_n I_{nf}$$

$$\frac{V_{dc}}{2}(S_{31} - S_{32}) = -Z_1 I_{cf} + V_{cn} + \frac{V_{dc}}{2}(S_{41} - S_{42}) + Z_n I_{nf} \quad (7.7)$$

In the mentioned figure by obeying KVL and KCL.

$$S_{11} + S_{12} = 1, S_{21} + S_{22} = 1, S_{31} + S_{32} = 1, S_{41} + S_{42} = 1. \quad (7.8)$$

Substituting (7.8) in (7.7) and simplifying

$$V_{an} = V_{dc}(S_{11} - S_{41}) - Z_1 I_{af} + Z_n I_{nf}$$

$$V_{bn} = V_{dc}(S_{21} - S_{41}) - Z_1 I_{bf} + Z_n I_{nf}$$

$$V_{cn} = V_{dc}(S_{31} - S_{41}) - Z_1 I_{cf} + Z_n I_{nf} \quad (7.9)$$

Set $Z_1 = r_f + L_f p$ and $Z_n = L_n p$ where $p = \frac{d}{dt}$. The output voltage equations can be

written as.

$$\begin{aligned} V_{an} &= V_{dc}(S_{11} - S_{41}) - r_f I_{af} - L_s p I_{af} + L_n p I_{nf} \\ V_{bn} &= V_{dc}(S_{21} - S_{41}) - r_f I_{bf} - L_s p I_{bf} + L_n p I_{nf} \\ V_{cn} &= V_{dc}(S_{31} - S_{41}) - r_f I_{cf} - L_s p I_{cf} + L_n p I_{nf}. \end{aligned} \quad (7.10)$$

$$\text{Define } S_a = S_{11} - S_{41}, S_b = S_{21} - S_{41}, S_c = S_{31} - S_{41}. \quad (7.11)$$

The output voltage equations can be written as

$$\begin{aligned} V_{an} &= V_{dc} S_a - r_f I_{af} - L_f p I_{af} + L_n p I_{nf} \\ V_{bn} &= V_{dc} S_b - r_f I_{bf} - L_s p I_{bf} + L_n p I_{nf} \\ V_{cn} &= V_{dc} S_c - r_s I_{cf} - L_s p I_{cf} + L_n p I_{nf}. \end{aligned} \quad (7.12)$$

The current flowing into the filter capacitor is given by

$$\begin{aligned} CpV_{an} &= I_{as} - I_{af} - I_{al} \\ CpV_{bn} &= I_{bs} - I_{bf} - I_{bl} \\ CpV_{cn} &= I_c - I_{cf} - I_{cl}. \end{aligned} \quad (7.13)$$

The neutral current is given by

$$I_{af} + I_{bf} + I_{cf} - I_{nf} = 0. \quad (7.14)$$

The DC link current flowing into the inverter is given as

$$I_d = S_{11} I_{af} + S_{21} I_{bf} + S_{31} I_{cf} - S_{41} I_{nf} \quad (7.15)$$

from above equation

$$I_d = (S_{11} - S_{41}) I_{af} + (S_{21} - S_{41}) I_{bf} + (S_{31} - S_{41}) I_{cf}$$

as previously defined,

$$I_{dc} = C_{dc} p V_{dc} = S_a I_{af} + S_b I_{bf} + S_c I_{cf}. \quad (7.16)$$

7.2 Modeling in q-d-o Stationary Reference Frame.

The q-d-o modeling of the four legged inverter in stationary reference frame is done by applying the transformation on Equations (7.1), (7.12), (7.13), and (7.16).

Consider (7.1).

$$\begin{bmatrix} V_{as} \\ V_{bs} \\ V_{cs} \end{bmatrix} = r_s \begin{bmatrix} I_{as} \\ I_{bs} \\ I_{cs} \end{bmatrix} - L_s p \begin{bmatrix} I_{as} \\ I_{bs} \\ I_{cs} \end{bmatrix} + \begin{bmatrix} V_{an} \\ V_{bn} \\ V_{cn} \end{bmatrix}. \quad (7.17)$$

Consider (7.12).

$$\begin{bmatrix} V_{an} \\ V_{bn} \\ V_{cn} \end{bmatrix} = V_{dc} \begin{bmatrix} S_a \\ S_b \\ S_c \end{bmatrix} - r_f \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} - L_f p \begin{bmatrix} I_{af} \\ I_{bf} \\ I_{cf} \end{bmatrix} + L_n p \begin{bmatrix} I_{nf} \\ I_{nf} \\ I_{nf} \end{bmatrix}. \quad (7.18)$$

The q-d-o quantities can be obtained from the a-b-c coordinates through following relationship.

$$[f_{qdos}] = T(\theta)[f_{abc}] \text{ where} \quad (7.19)$$

$$f_q = \frac{1}{3}(2f_a - f_b - f_c)$$

$$f_d = \frac{1}{\sqrt{3}}(f_b - f_c)$$

$$f_o = \frac{1}{3}(f_a + f_b + f_c).$$

The relations that will be used during the transformation are derived as below using (7.18) and (7.19).

$$\begin{bmatrix} V_{qdo} \end{bmatrix} = T(\theta) \begin{bmatrix} V_{aabbccn} \end{bmatrix}, \quad (7.20)$$

$$\begin{bmatrix} I_{qdos} \end{bmatrix} = T(\theta) \begin{bmatrix} I_{abc} \end{bmatrix}, \quad (7.21)$$

$$\begin{bmatrix} I_{qdof} \end{bmatrix} = T(\theta) \begin{bmatrix} I_{abcf} \end{bmatrix}$$

$$\begin{bmatrix} S_{qdos} \end{bmatrix} = T(\theta) \begin{bmatrix} S_{abc} \end{bmatrix}, \quad (7.22)$$

$$\begin{bmatrix} I_{qdon} \end{bmatrix} = T(\theta) \begin{bmatrix} I_{nnn} \end{bmatrix}, \quad (7.23)$$

also the inverse transformation can be applied as

$$f_{abc} = T^{-1}(\theta) f_{qdo}. \quad (7.24)$$

To express the average times obtained from Table 5.2 in terms of a-b-c voltages in Table 5.3, use the inverse transformation equations given by

$$f_a = f_q + f_o; f_b = \frac{-f_q}{2} - \frac{\sqrt{3}f_d}{2} + f_o; f_c = \frac{-f_q}{2} + \frac{\sqrt{3}f_d}{2} + f_o. \quad (7.25)$$

The transformed equations can be expressed as

$$\begin{bmatrix} V_{qs} \\ V_{ds} \\ V_{os} \end{bmatrix} = r_s \begin{bmatrix} I_{qs} \\ I_{ds} \\ I_{os} \end{bmatrix} - L_s p \begin{bmatrix} I_{qs} \\ I_{ds} \\ I_{os} \end{bmatrix} + \begin{bmatrix} V_{qn} \\ V_{dn} \\ V_{on} \end{bmatrix} \quad (7.26)$$

$$\begin{bmatrix} V_{qn} \\ V_{dn} \\ V_{on} \end{bmatrix} = V_{dc} \begin{bmatrix} S_q \\ S_d \\ S_o \end{bmatrix} - r_f \begin{bmatrix} I_{qf} \\ I_{df} \\ I_{of} \end{bmatrix} - L_f p \begin{bmatrix} I_{qf} \\ I_{df} \\ I_{of} \end{bmatrix} + L_n p \begin{bmatrix} 0 \\ 0 \\ 3I_{of} \end{bmatrix}. \quad (7.27)$$

The current flowing into the filter capacitor is given by

$$CpV_{qn} = I_{qs} - I_{qf} - I_{ql}$$

$$CpV_{dn} = I_{ds} - I_{df} - I_{dl}$$

$$CpV_{on} = I_{os} - I_{of} - I_{ol} \quad (7.28)$$

and

$$I_{dc} = C_{dc} pV_{dc} = \frac{3}{2}(S_q I_{qf} + S_d I_{df}) + 3S_o I_{of} \tag{7.29}$$

7.3 Controller Design for Current Regulation of Four Legged Active Power Filter.

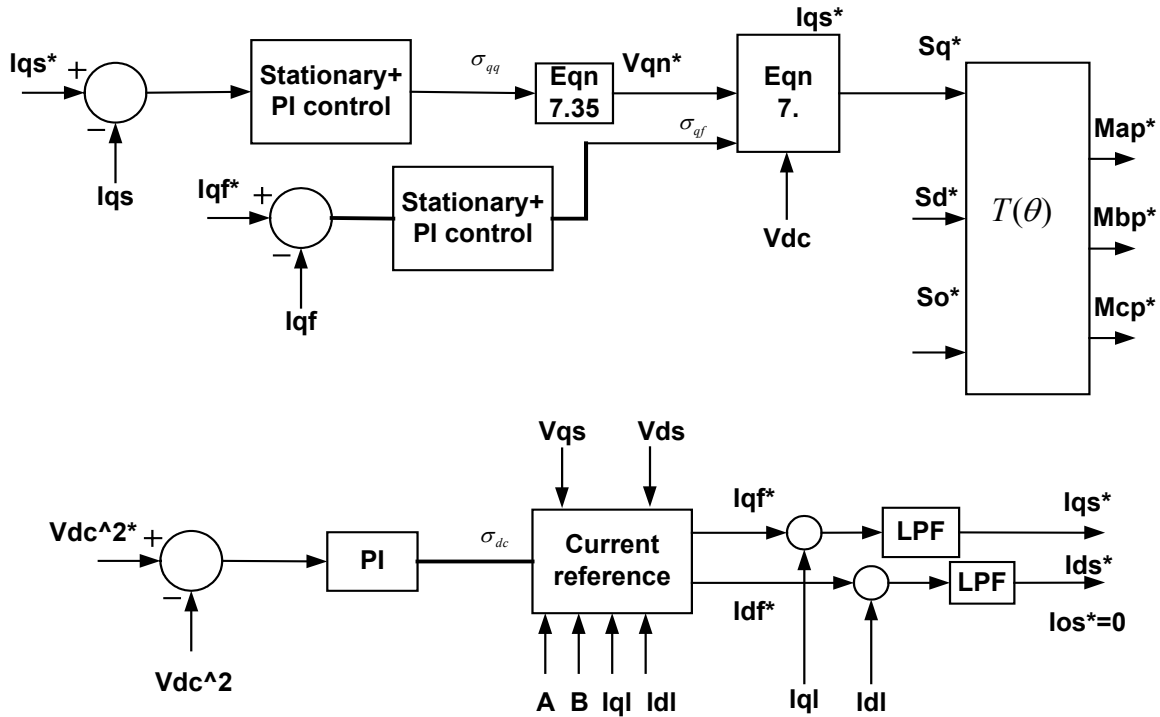


Figure 7.2 Controller structure for the four-leg inverter.

The controller objective is to regulate the three-phase currents from the source side for any load condition under unity power factor operation, in spite of its nature - balanced, unbalanced, and/or nonlinear. This control objective translates to the individual regulation of the three qdo source currents I_{qs} , I_{ds} , I_{os} in the stationary reference frame. A input-output linearization with decoupling control methodology is adopted [A.3]. This method linearizes the control equations, ensures system stability and high performance at all anticipated operating points given sufficient inverter DC voltage margin.

Using the source equations equations,

$$\begin{bmatrix} V_{qs} \\ V_{ds} \\ V_{os} \end{bmatrix} = r_s \begin{bmatrix} I_{qs} \\ I_{ds} \\ I_{os} \end{bmatrix} - L_s p \begin{bmatrix} I_{qs} \\ I_{ds} \\ I_{os} \end{bmatrix} + \begin{bmatrix} V_{qn} \\ V_{dn} \\ V_{on} \end{bmatrix} \quad (7.30)$$

and

$$\begin{bmatrix} V_{qn} \\ V_{dn} \\ V_{on} \end{bmatrix} = V_{dc} \begin{bmatrix} S_q \\ S_d \\ S_o \end{bmatrix} - r_f \begin{bmatrix} I_{qf} \\ I_{df} \\ I_{of} \end{bmatrix} - L_f p \begin{bmatrix} I_{qf} \\ I_{df} \\ I_{of} \end{bmatrix} + L_n p \begin{bmatrix} 0 \\ 0 \\ 3I_{of} \end{bmatrix} \quad (7.31)$$

$$CpV_{qn} = I_{qs} - I_{qf} - I_{ql}$$

$$CpV_{dn} = I_{ds} - I_{df} - I_{dl}$$

$$CpV_{on} = I_{os} - I_{of} - I_{ol} \quad (7.32)$$

and

$$I_{dc} = C_{dc} pV_{dc} = \frac{3}{2} (S_q I_{qf} + S_d I_{df}) + 3S_o I_{of}. \quad (7.33)$$

The required modulating signals need to be generated in order to achieve the desired goal. Hence from Equation (7.30)

$$r_s I_{qs} + L_s p I_{qs} = \sigma_{qq},$$

$$r_s I_{ds} + L_s p I_{ds} = \sigma_{dd},$$

$$r_s I_{os} + L_s p I_{os} = \sigma_{oo}. \quad (7.34)$$

Thus (7.31) can be as.

$$V_{qn}^* = V_{qs} - \sigma_{qq}$$

$$V_{dn}^* = V_{ds} - \sigma_{dd}$$

$$V_{on}^* = V_{os} - \sigma_{oo} \quad (7.35)$$

substituting

define

$$r_f I_{qf} + L_f p I_{qf} = \sigma_{qf},$$

$$r_f I_{df} + L_f p I_{df} = \sigma_{df},$$

$$r_f I_{of} + L_f p I_{of} = \sigma_{of} \quad (7.36)$$

substituting,

$$S_q = \frac{1}{V_{dc}} (V_{qn} - \sigma_{qf})$$

$$S_d = \frac{1}{V_{dc}} (V_{dn} - \sigma_{df})$$

$$S_o = \frac{1}{V_{dc}} (V_{qn} - \sigma_{of}) . \quad (7.37)$$

Consider DC link current equation,

$$I_{dc} = C_{dc} p V_{dc} = \frac{3}{2} (S_q I_{qf} + S_d I_{df} + 2S_o I_{of})$$

substituting S_q S_d and S_o

$$I_{dc} = C_{dc} p V_{dc} = \frac{3}{2V_{dc}} ((V_{qn} - \sigma_{qf}) I_{qf} + (V_{dn} - \sigma_{df}) I_{df} + 2(V_{qn} - \sigma_{of}) I_{of}) \quad (7.38)$$

$$\text{Let } A = V_{qn} - \sigma_{qf}$$

$$B = V_{dn} - \sigma_{df}$$

$$C = V_{on} - \sigma_{of} \quad (7.39)$$

Hence

$$I_{dc} = \frac{2}{3} V_{dc} C_{dc} p V_{dc} = AI_{qf} + BI_{df} + 2CI_{of}$$

thus

$$I_{dc} = \frac{1}{3} C_{dc} p V_{dc}^2 = AI_{qf} + BI_{df} + 2CI_{of}$$

Defining

$$C_{dc} p V_{dc}^2 = \sigma_{dc} ,$$

$$\frac{1}{3} \sigma_{dc} = AI_{qf} + BI_{df} + 2CI_{of} . \quad (7.40)$$

For unity power factor operation

$$Q = V_{qs} I_{ds} - V_{ds} I_{qs} + V_{os} I_{os} = 0 \quad (7.41)$$

but

$$I_{qs} = I_{qf} + I_{ql} , I_{ds} = I_{df} + I_{dl} \text{ and } I_{os} = I_{of} + I_{ol} .$$

Substituting

$$0 = V_{qs} (I_{qf} + I_{ql}) - V_{ds} (I_{df} + I_{dl}) + V_{os} (I_{of} + I_{ol})$$

Solving equation

$$I_{qf}^* = \frac{\frac{1}{3} \sigma_{dc} V_{qs} + V_{qs} AI_{ql} + 2V_{qs} BI_{dl} + BV_{ds} I_{ql} + 2CV_{qs} I_{ol} + V_{os} BI_{ol}}{AV_{qs} + BV_{ds}} \quad (7.42)$$

and

$$I_{df}^* = \frac{\frac{1}{3} \sigma_{dc} V_{ds} + 2V_{ds} AI_{ql} + V_{ds} BI_{dl} + AV_{qs} I_{dl} + 2CV_{ds} I_{ol} + V_{os} AI_{ol}}{AV_{qs} + BV_{ds}} \quad (7.43)$$

since the zero sequence current in the source must be zero therefore

$$I_{os}^* = I_{of} + I_{ol} = 0$$

hence

$$I_{of}^* = -I_{ol}$$

hence the reference currents for the source can be expressed as

$$I_{qs}^* = I_{qf}^* + I_{ql} \text{ and}$$

$$I_{ds}^* = I_{df}^* + I_{dl} . \quad (7.44)$$

7.3.1 Controller Definition for Reference Voltages

The transfer function for the feedback voltage over the desired voltage can be obtained as follows.

From (7.34)

$$CpV_{dc}^2 = \sigma_{dc} = K_{1q}(V_{dc}^{2*} - V_{dc}^2) - K_{2q}V_{dc}^2 .$$

Thus

$$V_{dc}^2(Cp + K_{1q} + K_{2q}) = K_{1q}V_{dc}^{2*} \text{ and hence the controller transfer function for q- axis is}$$

defined as

$$\frac{V_{dc}^2}{V_{dc}^{2*}} = \frac{K_{1q}}{(Cp + K_{1q} + K_{2q})}$$

where K_{1q} is the integral gain $= \frac{K_{1q}}{p}$ and K_{2q} is the proportional gain.

7.3.2 Definition for Reference Currents

The transfer function of the standard PI compensator in synchronous reference frame can be expressed as

$$G(s) = K_p + \frac{K_I}{s}.$$

As shown in [36] an equivalent stationary ac current controller achieves the same dc control response centered around the ac control frequency can be calculated as follows:

$$G(s) = \frac{1}{2}[g(s + j\omega) + g(s - j\omega)].$$

Substituting this expression in the above equation for synchronous frame controller.

$$G(s) = \frac{1}{2}\left[K_p + \frac{K_I}{s + j\omega} + K_p + \frac{K_I}{s - j\omega}\right]$$

$$G(s) = \frac{1}{2}\left[2K_p + \frac{2sK_I}{s^2 + \omega^2}\right]$$

$$G(s) = K_p + \frac{sK_I}{s^2 + \omega^2}.$$

A more practically realizable transfer function can be used is

$$G(s) = K_p + \frac{sK_I\omega_{cut}}{s^2 + 2\omega_{cut}s + \omega_o^2}$$

where ω_{cut} is the band cut off frequency and ω_o is the low cutoff frequency [36].

The transfer function for the feedback current over the reference current in stationary reference frame can be defined as follows

$$r_s I_{qs} + L_s p I_{qs} = \sigma_{qq} = \left(K_{pq} + \frac{sK_{Iq}\omega_{cut}}{s^2 + 2\omega_{cut}s + \omega_o^2}\right)(I_{qs}^* - I_{qs}).$$

Thus

$$(r_s + sL_s + K_{pq} + \frac{sK_{Iq}\omega_{cut}}{s^2 + 2\omega_{cut}s + \omega_o^2})I_{qs} = K_{pq} + \frac{sK_{Iq}\omega_{cut}}{s^2 + 2\omega_{cut}s + \omega_o^2}I_{qs}^*$$

Hence the input output transfer function can be given as.

$$\frac{I_{qs}}{I_{qs}^*} = \frac{K_{pq} + \frac{sK_{Iq}\omega_{cut}}{s^2 + 2\omega_{cut}s + \omega_o^2}}{(r_s + sL_s + K_{pq} + \frac{sK_{Iq}\omega_{cut}}{s^2 + 2\omega_{cut}s + \omega_o^2})}$$

which gives

$$\frac{I_{qs}}{I_{qs}^*} = \frac{(s^2 + 2\omega_{cut}s + \omega_o^2)K_{pq} + sK_{Iq}\omega_{cut}}{s^3L_s + s^2(r_s + K_{pq} + 2L_s\omega_{cut}) + s(2\omega_{cut}(r_s + K_{pq}) + \omega_o^2L_s + K_{Iq}\omega_{cut}) + (r_s + K_{pq})\omega_o^2}$$

where,

$$s^3L_s + s^2(r_s + K_{pq} + 2L_s\omega_{cut}) + s(2\omega_{cut}(r_s + K_{pq}) + \omega_o^2L_s + K_{Iq}\omega_{cut}) + (r_s + K_{pq})\omega_o^2 = 0$$

is the characteristic equation for the desired closed loop poles. Comparing it with the third ordered butterworth polynomial,

$$s^3 + 2s^2\omega_o + 2s\omega_o^2 + \omega_o^3 = 0$$

$$K_{pq} = 2\omega_oL_s - 2L_s\omega_{cut} - r_s \text{ and}$$

$$K_{Iq} = \frac{\omega_o^2L_s - 2\omega_{cut}(r_s + K_{pq})}{\omega_{cut}}$$

where K_{Iq} is the integral gain and K_{pq} is the proportional gain.

The d axis and o axis reference currents can be expressed as.

$$\frac{I_{ds}}{I_{ds}^*} = \frac{(s^2 + 2\omega_{cut}s + \omega_o^2)K_{pd} + sK_{Id}\omega_{cut}}{s^3L_s + s^2(r_s + K_{pd} + 2L_s\omega_{cut}) + s(2\omega_{cut}(r_s + K_{pd}) + \omega_o^2L_s + K_{Id}\omega_{cut}) + (r_s + K_{pd})\omega_o^2}$$

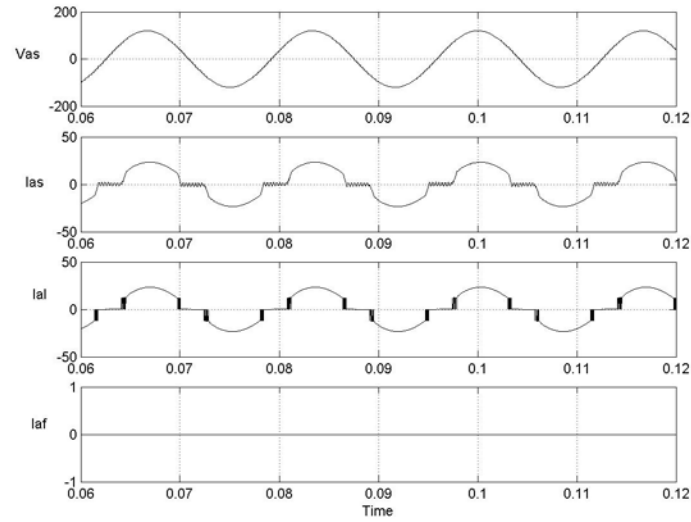
$$\frac{I_{os}}{I_{os}^*} = \frac{(s^2 + 2\omega_{cut}s + \omega_o^2)K_{po} + sK_{lo}\omega_{cut}}{s^3L_s + s^2(r_s + K_{po} + 2L_s\omega_{cut}) + s(2\omega_{cut}(r_s + K_{po}) + \omega_o^2L_s + K_{lo}\omega_{cut}) + (r_s + K_{po})\omega_o^2}.$$

The controller structure for the inverter is given in Figure 7.2. The manipulations in (7.36-7.44) ensures the reference current generated to compensate for the harmonics and unity power factor operation. The outputs of the three axis control loops are inverse-transformed to yield the fundamental components of the desired phase voltages, M_{ap}^* , M_{bp}^* and M_{cp}^* . These voltages are transformed to the stationary reference frame to determine the sector where the control voltage set lies and the identification of the expressions for the discontinuous modulation signals.

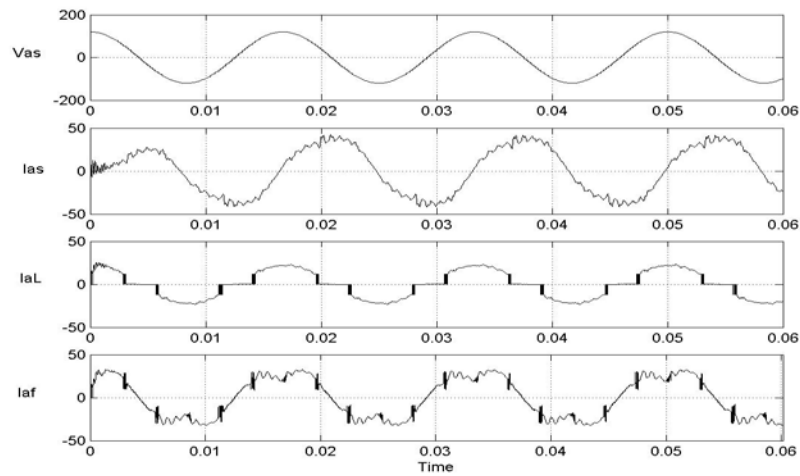
7.4 Simulation Results

Simulation results of the controlled four-leg active power filter feeding three phase nonlinear load. The line parameters are $r_s = 0.01 \text{ Ohm}$, $L_s = 0.5 \text{ mH}$, $V_{dc} = 300 \text{ V}$, and the filter parameters are $r_f = 0.1 \text{ Ohm}$, $L_f = 0.5 \text{ mH}$,.. The steady state phase load voltages given in Figure 6(a), track the references. Figure 7 gives the simulation results with three-single phase rectifiers connected on the three phase line . Figure 7.3 (a) Gives the simulation result without the active filter while (b)gives the result when the active filter in connected to the line. It can be seen that the line currents has harmonic contents when the active filter is absent. But when the active filter is switched on, the harmonics on the line are reduced. The phase shift in the source current and the voltage is generated due to the low pass filter in the control circuit. The phase shift is of 90 degrees, which can be compensated to achieve unity power factor operation.

Figure 7.4 shows the operation of the controller in tracking the reference source currents. It can be seen that the reference currents generated by the low pass filters are tracked exactly by the stationary PI controllers.



(a)



(b)

Figure 7.3 Waveforms for source voltage, source current, load current and active filter current for phase 'a' (a) Without active filter (b) With active filter

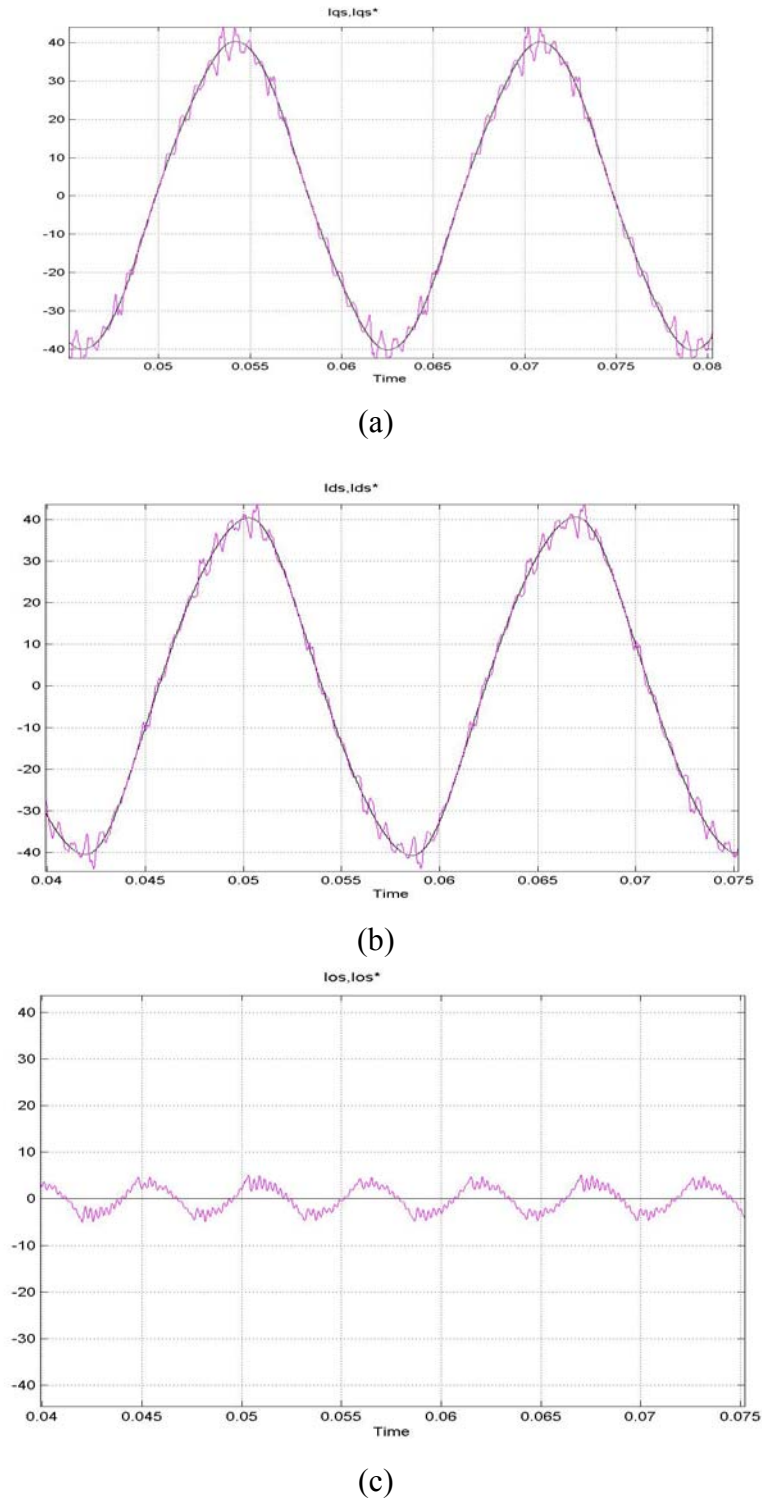


Figure 7.4 Waveforms for tracking of the q-d-o source currents with the stationary PI control. (a) q-axis (b) d-axis (c) o-axis